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Analysis of Variance

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Agenda

- variable types and statistical methods
- statistical tests: assumptions and procedures
- ANOVA: background and calculation (Excel)
- ANOVA: more backgr., typical designs, contrasts
- assumptions for using parametric tests (refresher)
- ANCOVA
- MANOVA and MANCOVA
- MANOVA: profile analysis

Categorical vs. continuous predct.

- categorical predictors (factors) contain a limited number of steps (e.g., male – female, experimentally manipulated or not)
- continuous have a (theoretically unlimited) number of steps (e.g., body height, weight, IQ)
- ANOVA (this session) is for categorical predictors, Regression analysis (next weeks session) is for continuous predictors

Categorical vs. continuous vars.

		Dependent variable	
		Categorical	Continuous
Independent variable	Categorical	X^2 test (chi-squared)	***t-test*** **ANOVA**
	Continuous	Logistic regression	***Correlation*** **Linear regression**

Relation vs. difference hypotheses

- relation hypotheses explore whether there is a relation between one (or more) independent and a dependent variable
- **difference hypotheses** explore whether there is a difference between the steps of one (or more) independent and a dependent variable
- the distinction between IV and DV is blurred for relation hypotheses
  → causality can only be inferred if the independent variable was experimentally manipulated

Within vs. between subject vars.

- within-subject variables are measures acquired from the same person (e.g., administering the same test before and after treatment; subtests / dimension of an IQ / personality test; EEG data) → idea that the "performance" or "properties" that characterize the person stay the same
- between-subjects variables are variables that distinguish between individuals (e.g, male-female)

Predictor and dependent variables

- independent = experimental = predictor variable, is a variable that is being experimentally manipu-lated in order to observe an effect
- dependent = outcome variable is the variable that is affected by the experimental manipulation

Questions?
Comments?

Assumptions of statistical tests

- population vs. sample ≈ parameter vs. statistic
  - ***population***: large group you want to make assumptions about vs. ***sample***: smaller group that you measure / observe (assuming to represent the population)
  - ***parameter***: «real» value in the population (e.g., population mean) vs. ***statistic***: (e.g., sample average)
- central limit theorem

Assumptions of statistical tests

- Standard error of mean – the more samples are taken from a population, the more exact the mean in the population can be described
→ imagine a series of dice throws (try it out)
- $s_{\bar{x}} = s / \sqrt{n}$

Assumptions of statistical tests

- H_0 – Null hypothesis (e.g., there is no group difference, the treatment doesn't work)
- H_1 – Alternative hypothesis
- Reject the H_0 (accept / retain H_1): observed difference is larger than exected by chance
- α-level (outer ends of the normal distribution)

Assumptions of statistical tests

- Distributions
  - z: position relative to mean in SDs (y – η) / σ
  - t: like z, but corrects for small samples
  - F: $\frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2} \sim F_{\nu_1,\nu_2}$ compares two variances (e.g., explained vs. unexplained)

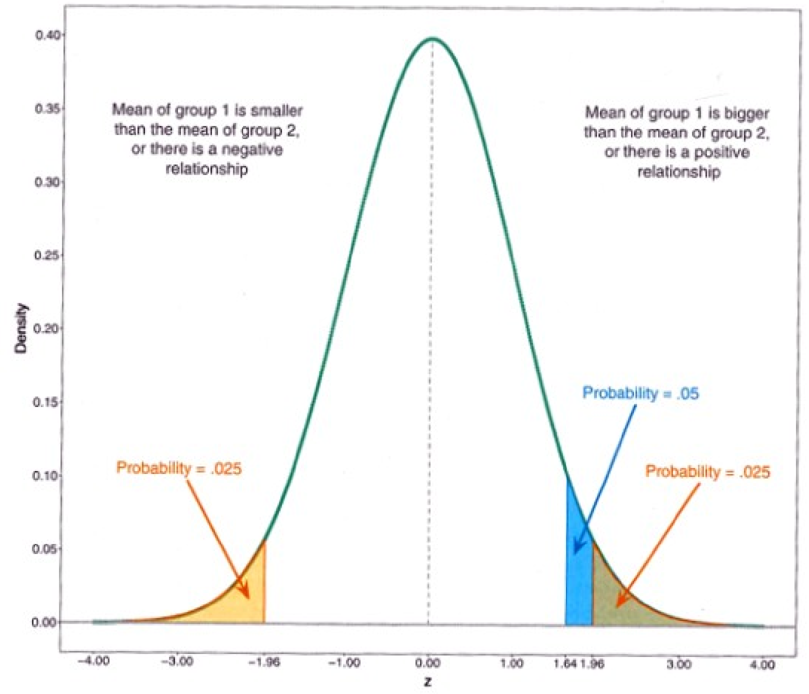
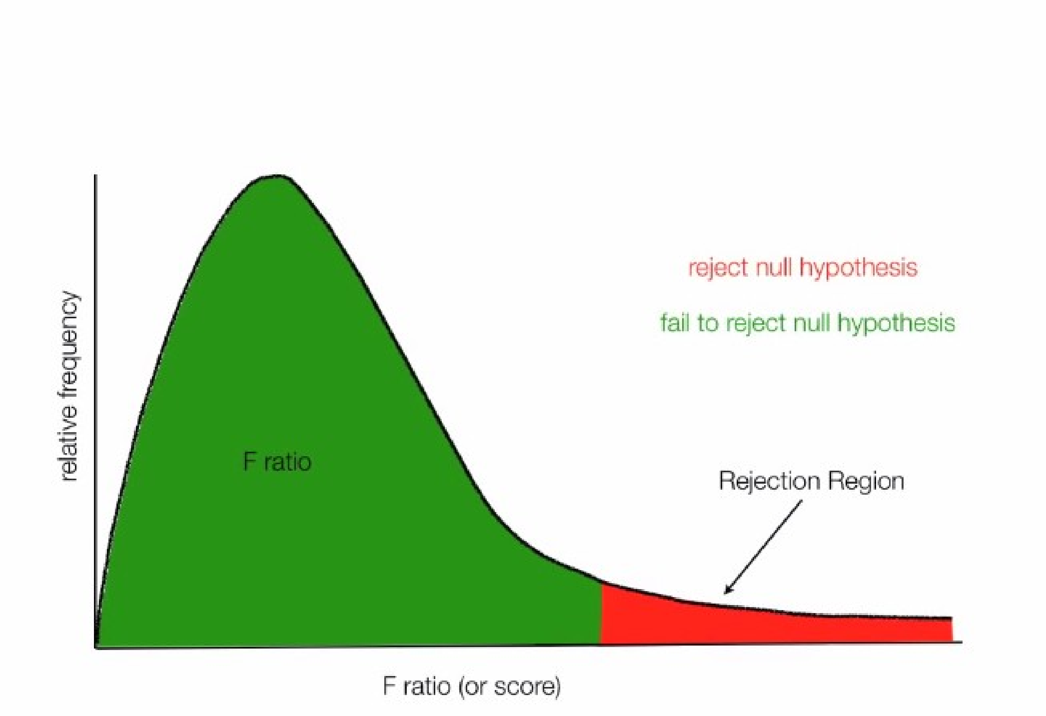


Figure 2.14 Diagram to show the difference between one- and two-tailed tests

Assumptions of statistical tests



Assumptions of statistical tests

- Type I error (False positive): one rejects the null hypothesis when it is true (α-probability).
- Type II error (False negative): one rejects the alternative hypothesis (fails to reject the null hypothesis) when the alternative hypothesis is true (β-probability).
- Usually deal with Type I errors; Type II errors are esp. important when determining sample size

Questions?
Comments?

Analysis of Variance

- compare two (or more) means to see whether they significantly differ from another
- evaluates the differences among means relative to the dispersion of the sampling distribution
  H_0: $\bar{Y}_1 = \bar{Y}_2 = \ldots = \bar{Y}_k$ ($\mu_1 = \mu_2 = \ldots = \mu_k$)

Analysis of variance

- WHAT WOULD BE THE BEST PREDICTOR VARIABLE FOR AN INDIVIDUAL MEASURE (E.G. BODY HEIGHT) IN A GROUP?
- WHY?
- HOW WOULD THIS CHANGE WITH INTRODUCING A FACTOR (E.G. SEX)?

Analysis of variance

- $y = b_0 + b_1 \cdot x_1 + \ldots + b_n \cdot x_n + e$

  $Y = BX + E$

  Y, y = dependent variable

  X, $[x_1 \ldots x_n]$ = predictor variable [0, 1]

  B, $[b_0 \ldots b_n]$ = predictor weights

  [group mean - sample mean]

  E, [e] = error term

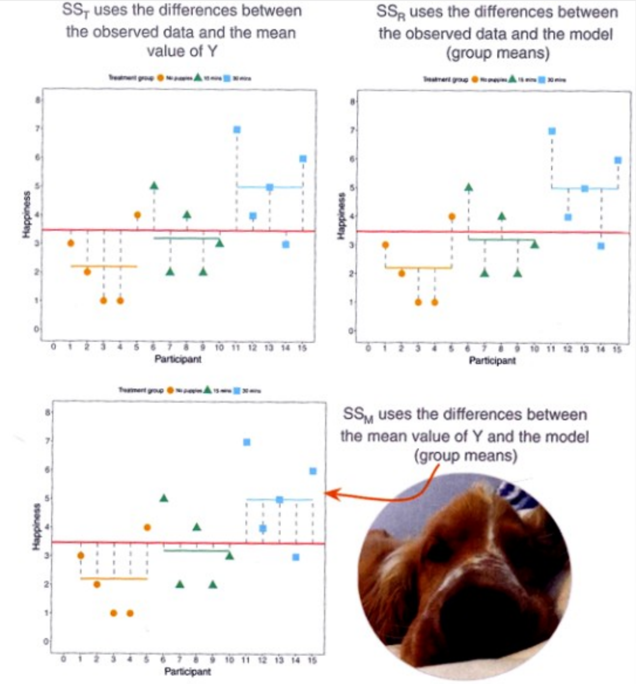


Figure 12.4 Graphical representation of the different sums of squares when comparing several means using a linear model. Also a picture of Ramsey as a puppy. Tufte would call him chartjunk, but I call him my adorable, crazy, spaniel

Analysis of variance

check out Analysis of Variance - Step-by-step.ods on MittUIB for details

- calculate group and sample mean (all groups)
- SS_R – calculate the difference between each individual value and its group mean and square it (SS of the *residuals*)
- SS_M – calculate the difference between group and sample mean, square it and multiply it by the number of group members (SS of the *model*)

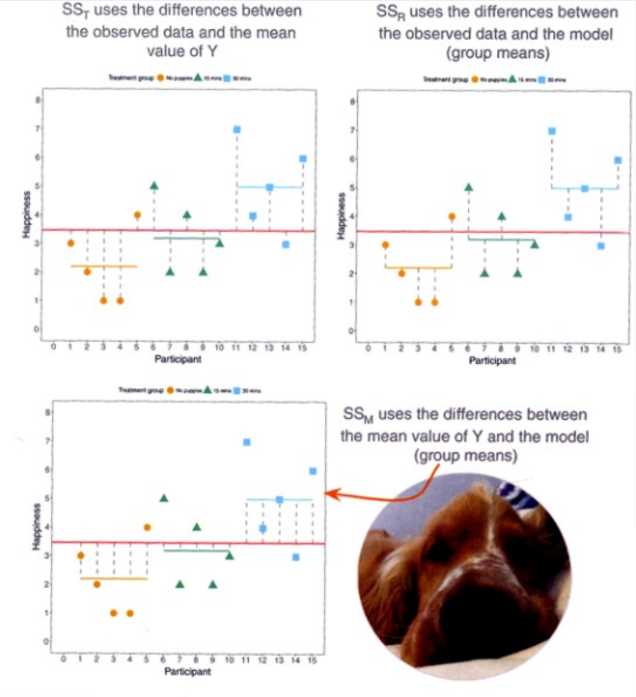


Figure 12.4 Graphical representation of the different sums of squares when comparing several means using a linear model. Also a picture of Ramsey as a puppy. Tufte would call him chartjunk, but I call him my adorable, crazy, spaniel

Analysis of variance

- MSS = SS / df
  (sum of squares / degrees of freedom)
- df_R = 15 (observations) – 3 (groups)
  df_M = 3 (groups) – 1
- MSS_R = 23,60 / 12 = 1,97
  MSS_M = 20,13 / 2 = 10,07
- $F_{(2,12)}$ = 10,07 / 1,97 = 5,12

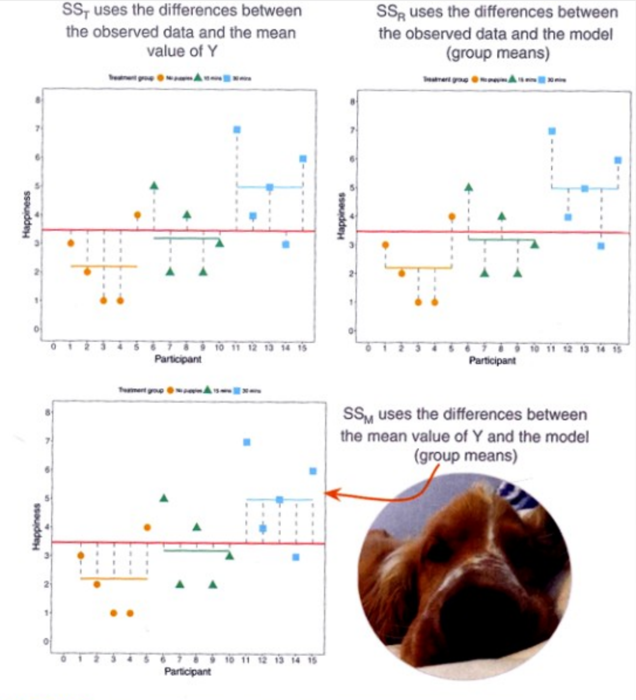
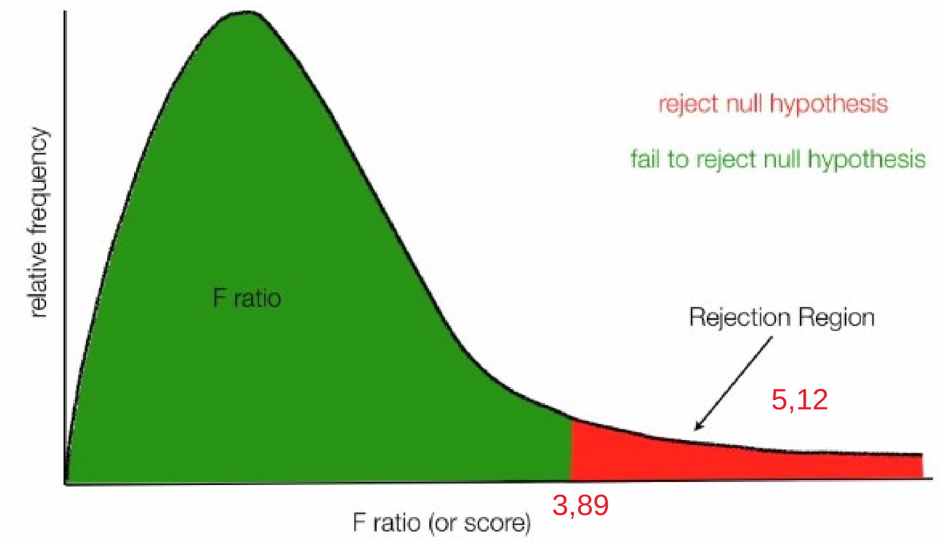


Figure 12.4 Graphical representation of the different sums of squares when comparing several means using a linear model. Also a picture of Ramsey as a puppy. Tufte would call him chartjunk, but I call him my adorable, crazy, spaniel

Analysis of variance



Questions?
Comments?

Analysis of Variance

- based upon two estimates / components of variance: (1) explained by differences in group means (effect) vs. (2) differences between group mean and individual score (error)

$Y_{ij} - GM = (Y_{ij} - \bar{Y}_j) + (\bar{Y}_j - GM)$

$$\sum_i \sum_j (Y_{ij} - GM)^2 = \sum_i \sum_j (Y_{ij} - \bar{Y}_j)^2 + n \sum_j (\bar{Y}_j - GM)^2$$

$SS_{total} = SS_{wg} + SS_{bg}$ $(df_{total} = df_{wg} + df_{bg})$

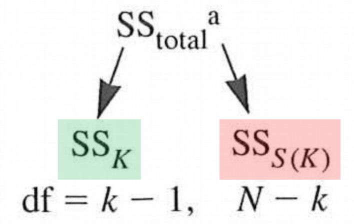
Analysis of Variance

- $df_{total} = N - 1$
  $df_{wg} = N - k$
  $df_{bg} = k - 1$
- $SS_{total} = SS_K + SS_{S(K)}$ (SS_K due to the k groups; $SS_{S(K)}$ due to subjects within the group)

Analysis of variance

- **one-way between-subjects ANOVA:**

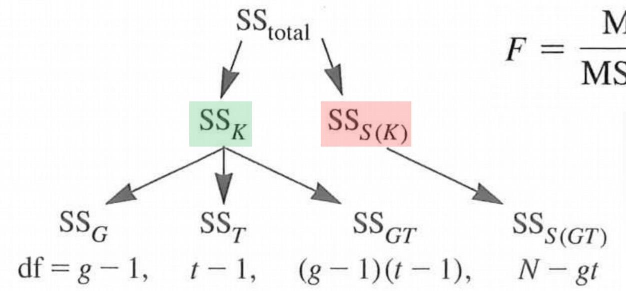
Treatment		
K_1	K_2	K_3
S_1	S_4	S_7
S_2	S_5	S_8
S_3	S_6	S_9



$$F = \frac{MS_K}{MS_{S(K)}} \qquad df = (k - 1), N - k$$

- **factorial between-subjects ANOVA**

		Teaching Techniques		
		T_1	T_2	T_3
Gender	G_1	S_1	S_5	S_9
		S_2	S_6	S_{10}
	G_2	S_3	S_7	S_{11}
		S_4	S_8	S_{12}

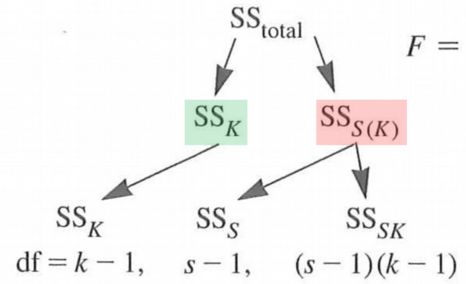


$$F = \frac{MS_G}{MS_{S(GT)}} \qquad df = (g - 1), N - gt$$

Analysis of variance

- **one-way within-subject ANOVA**

		Treatment		
		K_1	K_2	K_3
Subjects	S_1	S_1	S_1	S_1
	S_2	S_2	S_2	S_2
	S_3	S_3	S_3	S_3

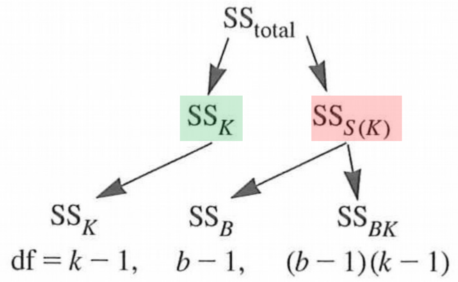


$$F = \frac{MS_K}{MS_{SK}} \qquad df = (k-1), (k-1)(s-1)$$

once individual differences are subtracted, the error term is usually smaller than in between-subject designs

- **one-way matched-randomized ANOVA**

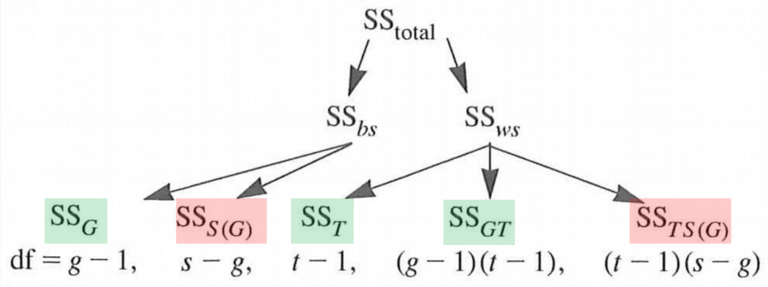
		Treatment		
		A_1	A_2	A_3
Blocks	B_1	S_1	S_2	S_3
	B_2	S_4	S_5	S_6
	B_3	S_7	S_8	S_9



subjects are matched on variable(s) highly related to the DV; per block b are as many subjects as factor steps k; should be more sens. than between-subject des.

Analysis of variance

- **mixed between-within-subjects ANOVA**

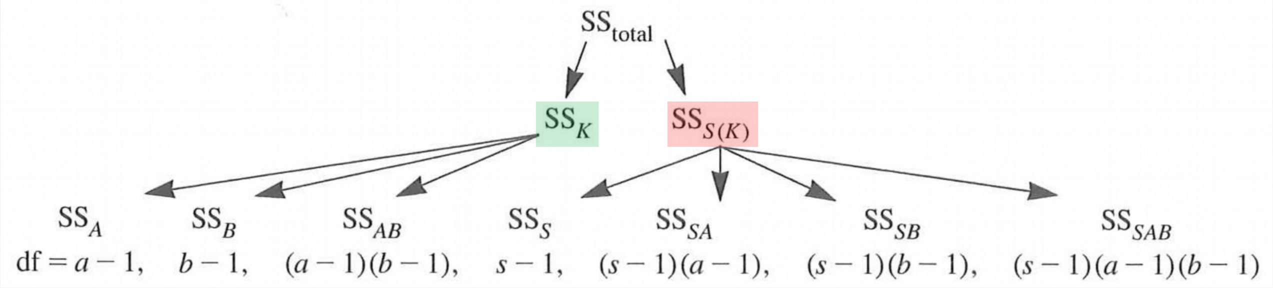


		Trials		
		T_1	T_2	T_3
Groups	G_1	S_1 S_2	S_1 S_2	S_1 S_2
	G_2	S_3 S_4	S_3 S_4	S_3 S_4

total SS is divided into a component attributable to the between-subjects part of the design (groups), another to the within-subject part (trials); each component is further partitioned into effects and errors; for all between-subjects, there is a single error term consisting of variance among subjects relative to each combination of between-subject IVs

Analysis of variance

- **factorial within-subject ANOVA**



		Treatment *A*		
		A_1	A_2	A_3
Treatment *B*	B_1	S_1 S_2	S_1 S_2	S_1 S_2
	B_2	S_1 S_2	S_1 S_2	S_1 S_2

$$F = \frac{MS_A}{MS_{SA}} \quad df = (a-1), (a-1)(s-1)$$

$$F = \frac{MS_B}{MS_{SB}} \quad df = (b-1), (b-1)(s-1)$$

$$F = \frac{MS_{AB}}{MS_{SAB}} \quad df = (a-1)(b-1), (a-1)(b-1)(s-1)$$

Analysis of Variance

design complexity:

- in between-subject designs subjects are nested to one level of IV or one combination of IVs
  (example: one teaching methods assigned to a classroom; children can't be randomly assigned)
- latin-square designs: to counter the effects of increasing experience, time of day, etc.

(a) Nested Designs

Teaching Techniques		
T_1	T_2	T_3
Classroom 1	Classroom 2	Classroom 3
Classroom 4	Classroom 5	Classroom 6
Classroom 7	Classroom 8	Classroom 9

(b) Latin-Square Designs[a]

		Order		
		1	2	3
Subjects	S_1	A_2	A_1	A_3
	S_2	A_1	A_3	A_2
	S_3	A_3	A_2	A_1

Analysis of Variance

contrasts:

- with factors with more than two levels or interactions → ambiguity; overall sign. but which difference «caused» the effect
- use contrasts to further investigate the difference
- *df*s as «non-renewable resource»
  → test most interesting comparisons at conventional α-levels
  → otherwise use Bonferroni-correct.
  → post-hoc compar. using Scheffé-adjust.
  $F' = (k - 1) \cdot F_{crit}$ (with k-1, df_{err})
- unequal N and non-orthogonality

	w_1	w_2	w_3
Comparison 1	1	−1	0
Comparison 2	1/2	1/2	−1
Comparison 3	1	0	−1

Analysis of Variance

fixed and random effects:

- fixed: selected levels of the IV
- random: sampling random levels of an (continouos) IV (e.g, word familiarity)

parameter estimates:

- sample means are unbiased estimators of population means but with a degree of uncertainty (SEM → confidence intervals)

Analysis of Variance

effect size measures:

indicate to which degree IV(s) and DV are related (variance in the DV that is predictable from IVs)

$\eta^2 = SS_{effect} / SS_{total}$

$\eta^2_p = SS_{effect} / (SS_{effect} + SS_{error})$

$\omega^2 \hat{=} (SS_{effect} - df_{effect} \cdot MS_{error}) / (SS_{total} + MS_{error})$

η^2 is flawed: (1) depends on number and sign. of other IVs in the design - proportion explained by any one variable will automatically decrease (→ partial η^2); (2) describes systematic / explained variance in a sample, but overestimates it in the population (esp. with small Ns → ω^2)

see: https://daniellakens.blogspot.com/2015/06/why-you-should-use-omega-squared.html

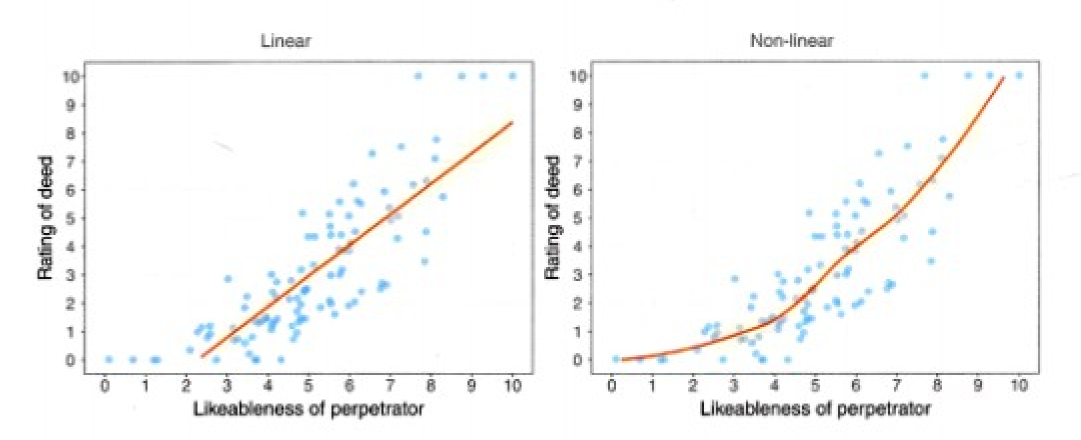
Questions?
Comments?

Parametric vs. non-parametric

- conditions for using parametric tests (such as correlation, regression, t-test, ANOVA)
- if one of these conditions is violated, non-parametric tests have to be used
- robustness against a violation of assumptions (most parametric tests are relatively robust against deviation from normality)

Parametric vs. non-parametric

- linearity (although the ANOVA is more robust against violations of this assumption than a regress.)



Parametric vs. non-parametric

- homogeneity of variance = homoscedasticity

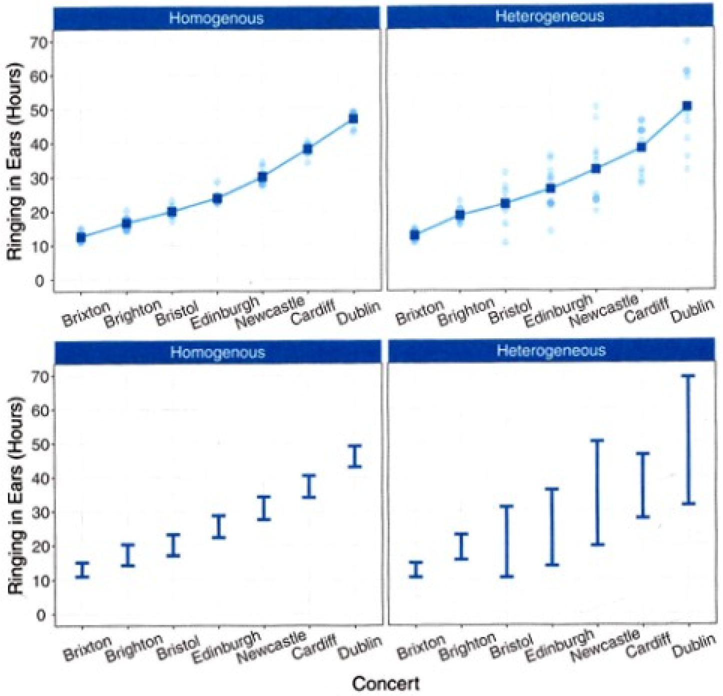
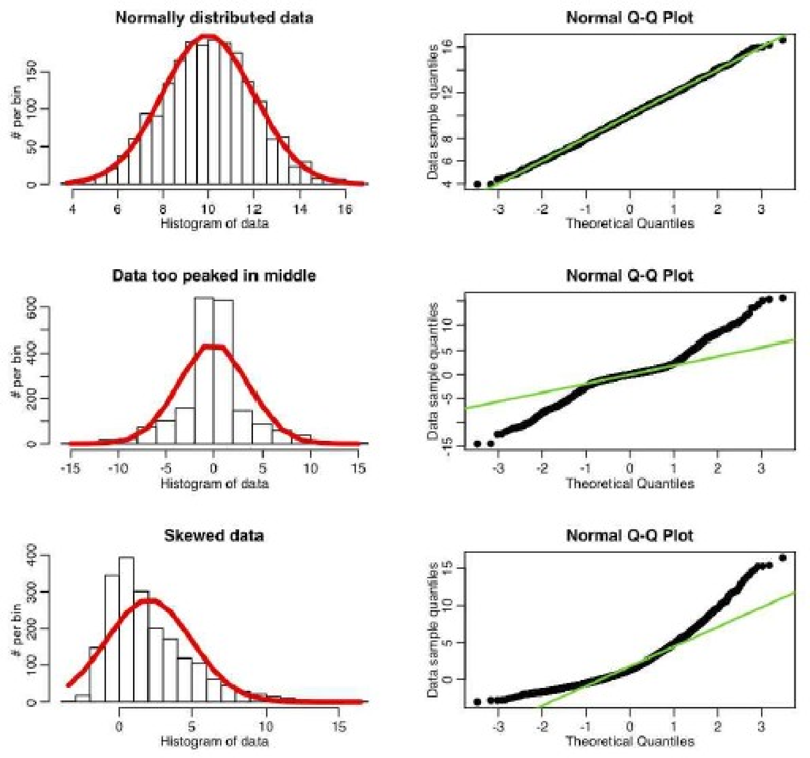


Figure 6.7 Graphs illustrating data with homogeneous (left) and heterogeneous (right) variances

Parametric vs. non-parametric

- normality and possible causes for normality violations



Checking assumptions

- linearity (for continuous predictors [ANCOVA]; scatterplot for predictor and dependent variable)
- normality
  - explorative data analysis: Box-Whisker plots for different factor stages, Normality plots
  - K-S-test for normality (within factor-steps)
- homogeneity of variances usually within tests or post-hoc (predictors vs. residuals)

Checking for outliers

- univariate – SPSS FREQUENCIES (box plots; for $N < 1000$ → $p = .001$ → $z = \pm 3.3$; only for DV and IVs that are used as covariates)
- multivariate: SPSS REGRESSION (Save → Distances → Mahalanobis; calculate "SIG.CHISQ(MAH_1,3)" and exclude $p < .001$; only for DV and IVs as covariates)
- IQR = Q3 – Q1 (sort your variable, take 25% position [Q1] and 75% position [Q3])
  Outlier: Q1 – IQR * 1.5 [liberal] / 3.0 [strict]
  Q3 + IQR * 1.5 [liberal] / 3.0 [strict]

Questions?
Comments?

ANCOVA

- extension of the ANOVA where main effects and interactions of IVs are adjusted for differences associated with one or more CV
- major purposes:
  (1) increase the sensitivity for the main effects by reducing the error term (reduce «undesirable» variance);
  (2) adjust the DV as if all participants were the same on the CV (statistical «matching» samples);
  (3) assess a DV after adjustment for other DVs (treated as CVs; autom. in MANOVA)
- variance partitioned: between groups (IVs), within group (CV)
  regression of CVs → DV, ANOVA of the IVs on the residuals

ANCOVA

research questions:

- explore main effects and interactions of Ivs, compare them using contrasts or trend analysis (same as ANOVA; while holding constant prior difference on a CV)
- evaluate the effect of CVs by assessing their expained variance
- evaluate the effect size of the IV after adj. for CVs

ANCOVA

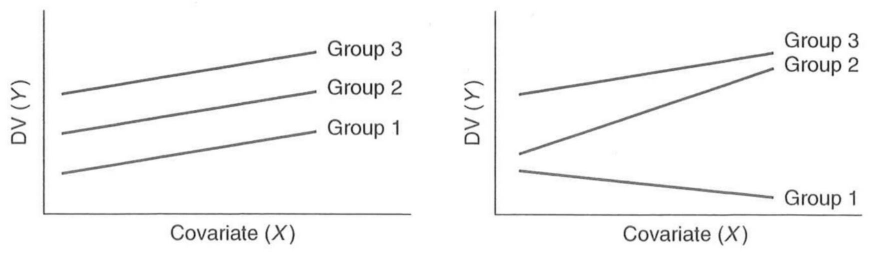
theoretical limitations:

- choose a small number of CVs (highly correlated with DV but not correlated with other Cvs)
- CVs must be independent of treatment (gathered before)
- adjusting mean DV score doesn't represent a «real-world»-situation

ANCOVA

practical issues:

- reliability of CVs (r_{XX} > .8)
- sufficient sample size per cell (level of IVs)
- absence of multicollinearity and singularity (SMC > .5 ~ redundant)
- linearity between CVs and between CVs and DV
- homogeneity of regression
-



(a) Homogeneity of regression (slopes) (b) Heterogeneity of regression (slopes)

ANCOVA

fundamental equations:

	Groups					
	Treatment 1		Treatment 2		Control	
	Pre	*Post*	*Pre*	*Post*	*Pre*	*Post*
	85	100	86	92	90	95
	80	98	82	99	87	80
	92	105	95	108	78	82
Sums	257	303	263	299	255	257

```
D = [1,  85, 100; ...
     1,  80,  98; ...
     1,  92, 105; ...
     2,  86,  92; ...
     2,  82,  99; ...
     2,  95, 108; ...
     3,  90,  95; ...
     3,  87,  80; ...
     3,  78,  82]

S1 = sum(D(D(:, 1) == 1, 2:3))
S2 = sum(D(D(:, 1) == 2, 2:3))
S3 = sum(D(D(:, 1) == 3, 2:3))

SB = [S1(1), S2(1), S3(1)]
SA = [S1(2), S2(2), S3(2)]
```

```
SSbg  = sum(SA .^ 2) / 3  - sum(SA) .^ 2 / 9
SSwg  = sum(D(:, 3) .^ 2) - sum(SA .^ 2) / 3

SSbgx = sum(SB .^ 2) / 3  - sum(SB) .^ 2 / 9
SSwgx = sum(D(:, 2) .^ 2) - sum(SB .^ 2) / 3

SPbg  = SA * SB' / 3 - sum(SA) * sum(SB) / 9
SPwg  = D(:, 2)' * D(:, 3) - SA * SB' / 3

SStbg = SSbg - ((SPbg + SPwg) ^ 2 / ...
        (SSbgx + SSwgx) - SPwg ^ 2 / SSwgx)
SStwg = SSwg - SPwg ^ 2 / SSwgx

Fcv = (SStbg / 2) / (SStwg / 5)
1 - fcdf(Fcv, 2, 5)
Fiv = (SSbg  / 2) / (SSwg  / 6)
1 - fcdf(Fiv, 2, 6)
etap_iv  = SStbg / (SStbg + SStwg)
```

ANCOVA

important issues:

- optimal set of CVs – weighed against loss in dfs, «power loss» if CVs are substantially correlated
- CVs are predictors in a sequential regr. perspect. (but multiple CVs er entered at once – std. regr.)
- testing for homogeneity of regression

```
MANOVA
   POST BY TREATMNT(1, 3) WITH PRE
   /PRINT=SIGNIF(BRIEF)
   /ANALYSIS = POST
   /METHOD=SEQUENTIAL
   /DESIGN PRE TREATMNT PRE BY TREATMNT.
```

ANCOVA

design complexity:

- a CV that is measured only once does not provide adjustment for within-subject effects
- adjustment for interactions of CV(s) and IV(s)
  no adjustm. (SPSS MANOVA), adj. (SPSS GLM)
- different CVs for the levels of IVs (imposs. in SPSS)

ANCOVA

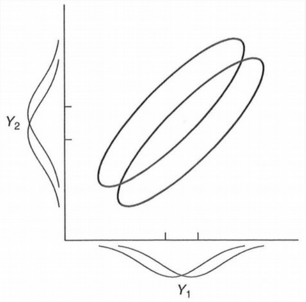
design alternatives:

- use differences (change scores) instead the pretest as CV or implement it as within-IV
- problem of change scores and floor or ceiling eff.
- problems with insufficient reliability
- blocking (dichotomize a CV: low, medium, high) or randomized blocks (k particip. per block)
  → does not need linearity, even works for curvilin.

Questions?
Comments?

MANOVA and MANCOVA

- generalization of the ANOVA for the combination of several DVs – statistically identical to linear discriminant analysis (MANOVA emphasizes whether multivar. differences are larger than chance; LDA emphasizes prediction, reliable separating groups by a multivariate combination / pattern)
- different linear combinations of DVs are formed for all main effects and interactions
- protection against inflation of type-I-error
- may reveal difference that don't show in UniANOVA
- avoids sphericity violations in univ. rep.-meas. ANOVA
- MANCOVA: simult. correcting for differences in covariates



MANOVA and MANCOVA

assumptions:

- multivariate normality
- absence of outliers (uni- and multivariate)
- homegeneity of variance-covariance matrices
- linearity
- homogeneity of regression (for MANCOVA)
- reliability of covariates
- absence of multicollinearity and singularity

MANOVA and MANCOVA

fundamental equations and calculation:

```
DL = [1, 1, 115, 108, 110; ...
      1, 2, 100, 105, 115; ...
      1, 3,  89,  78,  99; ...
      1, 1,  98, 105, 102; ...
      1, 2, 105,  95,  98; ...
      1, 3, 100,  85, 102; ...
      1, 1, 107,  98, 100; ...
      1, 2,  95,  98, 100; ...
      1, 3,  90,  95, 100; ...
      0, 1,  90,  92, 108; ...
      0, 2,  70,  80, 100; ...
      0, 3,  65,  62, 101; ...
      0, 1,  85,  95, 115; ...
      0, 2,  85,  68,  99; ...
      0, 3,  80,  70,  95; ...
      0, 1,  80,  81,  95; ...
      0, 2,  78,  82, 105; ...
      0, 3,  72,  73, 102]

GM = mean(DL(:, 3:4));
T  = zeros(2, 2); % treatment
D  = zeros(2, 2); % disability
DT = zeros(2, 2); % interaction
```

```
for ZT = 0:1
   T = T + (mean(DL(DL(:, 1) == ZT, 3:4)) - GM)' * ...
           (mean(DL(DL(:, 1) == ZT, 3:4)) - GM)  * nnz(DL(:, 1) == ZT);
end
for ZD = 1:3
   D = D + (mean(DL(DL(:, 2) == ZD, 3:4)) - GM)' * ...
           (mean(DL(DL(:, 2) == ZD, 3:4)) - GM)  * nnz(DL(:, 2) == ZD);
end
for ZI = 1:6
   DT = DT + (mean(DL(DL(:, 1) * 3 + DL(:, 2) == ZI, 3:4)) - GM)' * ...
             (mean(DL(DL(:, 1) * 3 + DL(:, 2) == ZI, 3:4)) - GM)  * ...
             nnz(DL(:, 1) * 3 + DL(:, 2) == ZI);
end
DT = DT - T - D
E = (DL(:, 3:4) - GM)' * (DL(:, 3:4) - GM) - D - T - DT
% determininants (det) as the matrix analogue of variance
LT  = det(E) / det(T  + E)
LD  = det(E) / det(D  + E)
LDT = det(E) / det(DT + E)
FT  = ((1 - LT  ^ (1/1)) / LT  ^ (1/1)) * (11 / 2)
FD  = ((1 - LD  ^ (1/2)) / LD  ^ (1/2)) * (22 / 4)
FDT = ((1 - LDT ^ (1/2)) / LDT ^ (1/2)) * (22 / 4)
ST  = 1 - fcdf(FT,  2, 11)
SD  = 1 - fcdf(FD,  4, 22)
SDT = 1 - fcdf(FDT, 4, 22)
```

MANOVA and MANCOVA

applicability:

- MANOVA works best with highly negatively correlated DVs and acceptably with moderately (pos. or neg.) correlated Dvs; wasteful if very highly pos. related (no improved prediction) or uncorrelated (no advant. over ANOVA)

MANOVA and MANCOVA

statistical inference (Wilks Λ, Hotelling, Pillai, Roy's gcr):

- identical for factors with two levels
- for more than two levels: Wilks, Hotelling, Pillai pool dimensions, Roy considers first dimension / contrast
- Wilks: likelihood statistics for equal population mean vectors vs. group mean vectors in the sample
  Hotelling: pooled ratio of effect to error variance
  Pillai: pooled effect variances
- Wilks, Hotelling, Roy: most robust if strongest contrib. fr. first contr.
- Pillai more robust (against small sample sizes, inhomog. of var.)
- → use Wilks unless there is reason to use Pillai

MANOVA and MANCOVA

strategies for assessing DVs:

- if DVs are uncorrelated UniANOVA is acceptable
- if DVs are correlated, use stepdown analysis (analogue to sequential regression) in combination with UniANOVA and evaluate possible pattern:
  (1) sign. in UniANOVA, nonsign. stepdown → variance already explained by higher-order DVs
  (2) nonsign. in UniANOVA, sign. stepdown → DV takes on «importance» from higher-order DVs

Questions?
Comments?

MANOVA: Profile analysis

- special application of the MANOVA with several DVs measured on the same scale: (1) same DV over time (repeated measures), (2) several DVs (e.g., WISC-subtests) at the same time, (3) several DVs over time (doubly multivar. design) or (4) compare profiles of two groups (POMS, WISC, neuropsych. battery)

MANOVA: Profile analysis

typical research questions:

- testing parallelism of profiles through interaction (group × test)
- overall group performance differences
- flatness of profiles (lack of diff. between subtests)
- «typical» profiles for different groups (mean prof.)

MANOVA: Profile analysis

assumptions and limitations:

- N per factor level should be ≥ number of levels
- robust against unequal cell sizes and non-normal.
- for equal cell sizes, homogeneity of variance-covariance matr. doesn’t have to be evaluated
- extreme sensitivity to outliers
- non-linearity → loss of power for parallelism-test

MANOVA: Profile analysis

fundamental equations and calculation:

```
D = [1, 7, 10, 6, 5; ...
     1, 8,  9, 5, 7; ...
     1, 5, 10, 5, 8; ...
     1, 6, 10, 6, 8; ...
     1, 7,  8, 7, 9; ...
     2, 4,  4, 4, 4; ...
     2, 6,  4, 5, 3; ...
     2, 5,  5, 5, 6; ...
     2, 6,  6, 6, 7; ...
     2, 4,  5, 6, 5; ...
     3, 3,  1, 1, 2; ...
     3, 5,  3, 1, 5; ...
     3, 4,  2, 2, 5; ...
     3, 7,  1, 2, 4; ...
     3, 6,  3, 3, 3]

GM = mean(D(:,            2:5), 1)
M1 = mean(D(D(:, 1) == 1, 2:5), 1)
M2 = mean(D(D(:, 1) == 2, 2:5), 1)
M3 = mean(D(D(:, 1) == 3, 2:5), 1)
```

```
figure; hold on; xlim([0.5 4.5]);
plot([1 2 3 4], M1, 'r*-')
plot([1 2 3 4], M2, 'b*-')
plot([1 2 3 4], M3, 'k*-')

SSbg = 5 * 4 * ((mean(M1, 2) - mean(GM, 2)) ^ 2 + ...
                (mean(M2, 2) - mean(GM, 2)) ^ 2 + ...
                (mean(M3, 2) - mean(GM, 2)) ^ 2)
SSwg = 4 * sum((mean(D(:, 2:5), 2) -          ...
               [repmat(mean(M1, 2), 5, 1); ...
                repmat(mean(M2, 2), 5, 1); ...
                repmat(mean(M3, 2), 5, 1)]) .^ 2)
Fg = (SSbg / 2) / (SSwg / 12)
Sg = 1 - fcdf(Fg, 2, 12)

% calculate differences among tests / ratings
DD = [D(:, 1), -diff(D(:, 2:5), 1, 2)]
DGM = mean(DD(:,             2:4), 1)
DM1 = mean(DD(DD(:, 1) == 1, 2:4), 1)
DM2 = mean(DD(DD(:, 1) == 2, 2:4), 1)
DM3 = mean(DD(DD(:, 1) == 3, 2:4), 1)
```

MANOVA: Profile analysis

fundamental equations and calculation (cont.):

```
Swg = (DD(:, 2:4) - [repmat(DM1, 5, 1); repmat(DM2, 5, 1); repmat(DM3, 5, 1)])' * ...
      (DD(:, 2:4) - [repmat(DM1, 5, 1); repmat(DM2, 5, 1); repmat(DM3, 5, 1)])
Sbg = 5 * ((DM1 - DGM)' * (DM1 - DGM) + (DM2 - DGM)' * (DM2 - DGM) + ...
           (DM3 - DGM)' * (DM3 - DGM))

LP = det(Swg) / det(Swg + Sbg)
% it is not clear to me why the s in (1/s) is set to 2; however,
% the F value is numerically identical to the SAS output (p. 367)
FP = (1 - LP ^ (1/2)) / (LP ^ (1/2)) * (20 / 6)
SP = 1 - fcdf(FP, 6, 20)
etapP = 1 - LP ^ (1/2)

T2F = 15 * DGM * inv(Swg) * DGM'
FF = (15 - 3 - 4 + 2) / (4 - 1) * T2F
SF = 1 - fcdf(FF, 3, 10)
LF = 1 / (1 + T2F)
etapF = 1 - LF ^ (1 / 1)
```

NB: ***parallelism*** *is the H0, profiles are parallel if there are no group differences in profile*

NB: ***flatness*** *is also the H0, profiles are flat if there are no differences between scores within the profile*

MANOVA: Profile analysis

important issues:

- univariate repeated-measure analyses require sphericity (if more than two levels; for longitudinal studies, sphericity is unlikely; the assumption would be similar correl. between 5 to 6 vs. 5 to 10 years of age)
- univariate analyses: sphericity-correction using Greenhouse-Geisser, Huynh-Feldt
- multivariate analyses require larger samples
- best alternative: trend analysis (polynomial)
- linear discrim. analysis: classification of profiles

Questions?
Comments?

Summary

- variable types and statistical methods
- statistical tests: assumptions and procedures
- ANOVA: background and calculation (Excel)
- ANOVA: more backgr., typical designs, contrasts
- assumptions for using parametric tests (refresher)
- ANCOVA
- MANOVA and MANCOVA
- MANOVA: profile analysis

Literature

Tabachnik, B. G., Fidell, L. S. (2013). *Using Multivariate Statistics* (6th ed.). New York, NY: Pearson. (Ch. 3, 6, 7 & 8)

Field, A. (2017). *Discovering Statistics Using IBM SPSS Statistics*. London, UK: Sage Publications Ltd.

Thank you for your interest and your attention!

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